Journal of Manufacturing Systems
Vol. 20/No. 1
2001

Optimal Fixture Design for Drilling Through Deformable Plate Workpieces Part II: Results

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Abstract

This is the second of two manuscripts which employ finite element analysis and optimization algorithms to design optimal fixtures for drilling through deformable workpieces. Part I has addressed the problem formulation, suggesting five different objective functions that capture different geometric characteristics of the machined surface. The performance of the five obtained fixture layouts under different drilling conditions is studied in this paper. The drilling studies include single, sequential, and simultaneous (gang) drilling. These studies demonstrate the known fact that more frequent setups and longer machining time periods result in higher workpiece accuracies. The simulations also suggest that the proposed optimal fixturing model developed in this study can be used to command greater control over the drilling process, resulting in elevated drilling quality. The numerical value of the latter is captured by the objective function values. The computer simulations further identify optimal fixturing layouts that can be implemented in an industrial environment.

Keywords: *Simulation, Drilling, Deformable, Workpiece, Finite Elements, Fixture Design, Optimal*

1. Introduction

In this manuscript, the optimal fixture layouts obtained by minimizing five different objective functions presented in Part I are evaluated for four different drilling scenarios. In these evaluation studies, the fixturing problem is posed as a constrained optimization problem in which the physical fixture constraints define the problem domain, and the desired fixturing characteristics are optimized with respect to the selected objective function. Five different objective functions that capture different geometric characteristics of the machined surface have been suggested. The quadratic differences between the nominal and simulated radii and diameters, as they are summed over the entire digitized machine surface, are expressed as Δ_1 and Δ_3, respectively

$$\Delta_1 = \frac{1}{l.m.n.}\sum_{k=1}^{l}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(R_{nom_k} - R_{ijk}\right)^2 \quad (1)$$

$$\Delta_3 = \frac{1}{l.m.n.}\sum_{k=1}^{l}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(D_{nom_k} - D_{ijk}\right)^2 \quad (2)$$

Here, R_{ijk} and D_{ijk} represent the radial and diametrical measurements of a point (i,j) at the kth hole. R_{nom_k} and D_{nom_k} refer to the nominal radius and diameter of the kth hole, respectively. The maximum differences between the nominal and simulated radii and diameters are captured by the Δ_2 and Δ_4 functions, which are given by:

$$\Delta_2 = Max\left(R_{nom_k} - R_{ijk}\right)^2 \quad i = 1-n, j = 1-m, k = 1-l \quad (3)$$

$$\Delta_4 = Max\left(D_{nom_k} - D_{ijk}\right)^2 \quad i = 1-n, j = 1-m, k = 1-l \quad (4)$$

The function Δ_5 is a measure of the deviation of the machined surface from a perfect cylindrical hole:

$$\Delta_5 = \sum_{k=1}^{l}[\alpha\Theta_k + \beta\Psi_k + \gamma\Omega_k] \quad (5)$$

where

$$\Theta_k = \frac{1}{m.n}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(R_{lsq_j} - \sqrt{\left(X_{ij} - X_{o_j}\right)^2 + \left(Y_{ij} - Y_{o_j}\right)^2}\right)^2 \quad (6)$$

$$\Psi_k = \frac{1}{m}\sum_{j=1}^{m}\left[\left(X_{o_j} - \bar{X}_o\right)^2 + \left(Y_{o_j} - \bar{Y}_o\right)^2\right] \quad (7)$$

$$\Omega_k = \frac{1}{m}\sum_{j=1}^{m}\left(R_{lsq_j} - \bar{R}_{lsq}\right)^2 \quad (8)$$

Here, Θ_k measures the least-square fit of the m circles through the coordinates (X_{ij}, Y_{ij}) of points that reside on the machined surface. Ψ_k captures the deviations of the center coordinates, and Ω_k measures the radii variations of the m least-square circles. . In Eq. (5), α, β, and γ are design weight factors that are set by the user to reflect the circumstances at hand. In this study, α, β, and γ are set to be one unit. Before embarking on numerical computer simulations, relevant previous research reports are reviewed.

The utilization of finite element analysis and optimization algorithms for solving the fixturing problem is reported in Lee and Haynes,[1] Menassa and DeVries,[2] and Pong.[3] In these reports, optimization methods that are based on gradient techniques are used. These techniques are sensitive to the initial point selection and tend to converge to the first local minimum encountered. When utilizing optimization algorithms that are based on function evaluations,[4-7] or when applying gradient-based techniques,[8-10] if the objective function is multi-modal in the domain, the first minimum encountered is reported as the global minimum.

The fixture optimization simulations conducted in this paper utilize a Simulated Annealing (SA) algorithm as discussed in Corana et al.,[11] Romeo, Sangiovanni, and Sechen,[12] and White.[13] SA is based on random function evaluations that are reported to escape local minima. SA explores the entire function surface and tries to optimize the objective function while moving both uphill and downhill. Furthermore, SA does not require function continuity. Computer simulations that evaluate optimal fixtures when performing single, sequential, and simultaneous (gang) drilling operations are presented in the next section.

2. Computer Simulations

The finite element fixturing modeling, together with the material removal tools developed and the objective function analysis addressed in Part I, form the optimal fixturing model for drilling through plate deformable workpieces. The computer simulations presented in this section have been conducted with the aid of the optimal fixturing model. Before presenting the specifics of the simulations discussed in this section, it may be of importance to note that each simulation involves one of the above five objective functions as proposed in Part I and is conducted under a rather demanding iterative computational scheme. For example, the elastic plate is initially constrained consistently with the loading and geometric boundary conditions shown in Figure 1 of Part I. The associated boundary value problem put forth is then solved using the ABAQUS FE software.[14] The 3-D finite element solution is then used to extract the pertinent information regarding the deformed shape of the drilled hole as needed to evaluate the associated objective function. The restraining boundary conditions associated with the fixture locators at the lower surface of the workpiece are then perturbed consistent with a simulated annealing optimization scheme,[11] which is used to extract a global minimum for the selected objective function. In the simulations reported herein, objective function minima and associated optimal fixture configurations often required 2000 to 4000 iterations guided by the adopted simulated annealing optimization algorithm. Thus, each optimal fixture configuration reported in this study required approximately 30 to 48 hours of computing time on an R10000 SGI multiprocessor

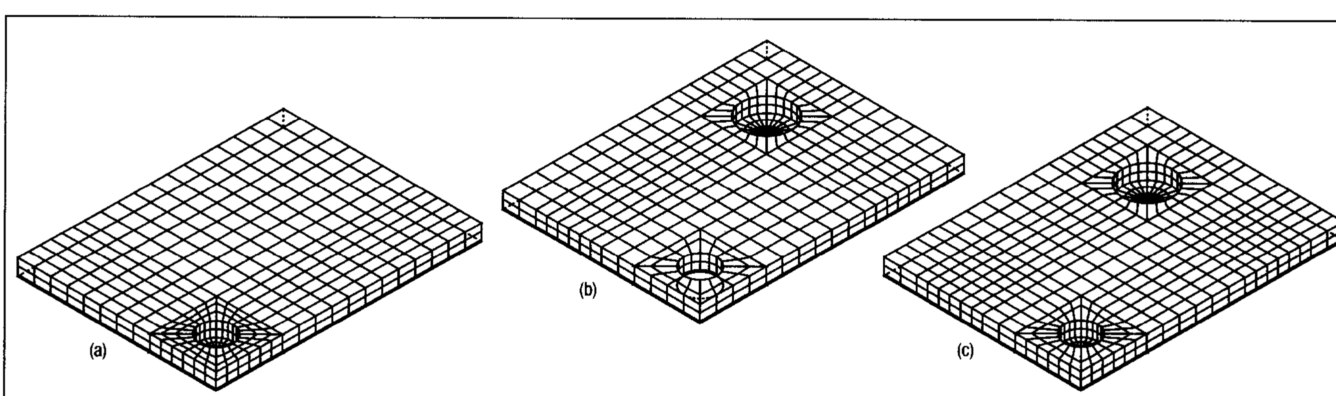


Figure 1
(a) FE mesh of 1/2 in. hole drilled in cases I and III. Mesh contains 936 20-noded isoparametric elements with 15573 degrees of freedom. (b) FE mesh of existing 1/2 in. hole and drilled 3/4 in. hole generated in cases II and III. Mesh contains 840 20-noded isoparametric elements with 14376 degrees of freedom. (c) FE mesh of gang drilling of 1/2 and 3/4 in. holes, case IV. Mesh contains 1104 20-noded isoparametric elements with 18681 degrees of freedom.

machine. Consistent with the above for a given geometry and drilling condition, a total computing time of about 150 hours was required to obtain the five different optimal fixture configurations associated with using the Δ_i, $i = 1 - 5$ objective functions.

To test the proposed fixturing formulations, computer simulation tests of four different drilling scenarios (cases I, II, III, and IV) are evaluated. The plate dimensions are depicted in Figure 2 of Part I, and the corresponding finite element meshes are shown in *Figure 1*. Case I seeks the optimal fixture for drilling a 1/2 in. hole centered at (2.5, 0.5) (in.). Case II determines the optimal fixture layout for drilling a 3/4 in. hole at (1.0, 3.0) (in.) in the presence of the 1/2 in. hole drilled in case I. In case III, the 1/2 and 3/4 in. holes drilled in cases I and II, respectively, are drilled using a single fixturing configuration. This results in a 50% reduction in the setup time, yet the machining accuracy needs to be evaluated. In case IV, the 1/2 and 3/4 holes are gang drilled. Here, the machining time is shortened, yet the workpiece is exposed to high loads, and the machined surface is expected to be of the lowest accuracy. In these simulations, an aluminum plate with an elastic modulus E = 1.0E+07 (psi), a Poisson ratio $\nu = 0.3$, with length, width, and thickness of 4, 3, and 1/4 in., respectively, is drilled by using the optimal fixturing configurations.

In case I, the optimal fixture is sought for drilling a 1/2 in. hole centered at $(X, Y) = (2.5, 0.5)$ in. The values of the simulated drilling thrust F_Z and torque M are 499 (lb) and 95 (lb-in.), respectively.[3] The positions of the locators and clamps of the five optimal fixtures, FIX1, FIX2, ..., FIX5, are depicted in

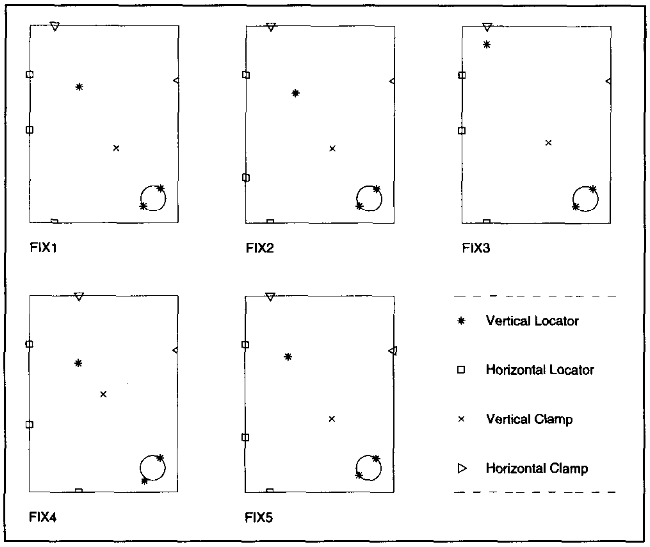


Figure 2
Optimal Fixture Configurations FIX1, FIX2, ..., FIX5 Generated Through Minimization of $\Delta_1, \Delta_2, ..., \Delta_5$, Respectively, for Drilling 1/2 in. Hole of Case I.

Figure 2. These fixtures are obtained by minimizing the objective functions Δ_1, Δ_2, ..., Δ_5, respectively. *Table 1* summarizes the optimal fixture parameters $\bar{u}$ of the five optimal fixture configurations. Notice that the minimization of the five objective functions results in small variations in the positions of the locators and clamps of the five optimal fixtures. The value of F_{Z_1} in FIX4 is, however, an exception. The latter fixture parameter is an order of magnitude smaller than the optimal values of the vertical clamps in FIX1, FIX2, FIX3, and FIX5.

Table 2 lists the objective function values Δ_i, i = 1–5 of the five fixture configurations FIXj, j = 1–5. Note that for FIXj the value of Δ_i is minimum

Table 1
Case I—Drilling 1/2 in. hole centered at (2.5, 0.5) (in.). Locator positions (in.) and clamping forces (lb) when Δ1 - Δ5 are minimized.

Optimization Variables	FIX1	FIX2	FIX3	FIX4	FIX5
L_1	(1.00, 2.75)	(1.00, 2.62)	(0.50, 3.62)	(1.00, 2.62)	(0.88, 2.75)
L_2	(2.30, 0.35)	(2.30, 0.35)	(2.30, 0.35)	(2.33, 0.23)	(2.30, 0.35)
L_3	(2.65, 0.70)	(2.65, 0.70)	(2.65, 0.70)	(2.65, 0.70)	(2.65, 0.70)
L_4	(0.00, 3.00)	(0.00, 3.00)	(0.00, 3.00)	(0.00, 3.00)	(0.00, 3.00)
L_5	(0.00, 1.88)	(0.00, 0.92)	(0.00, 1.88)	(0.00, 1.38)	(0.00, 1.12)
L_6	(0.50, 0.00)	(0.50, 0.00)	(0.50, 0.00)	(1.00, 0.00)	(0.50, 0.00)
C_1	(1.75, 1.50)	(1.75, 1.50)	(1.75, 1.62)	(1.50, 2.00)	(1.75, 1.50)
C_2	(3.00, 2.88)	(3.00, 2.88)	(3.00, 2.88)	(3.00, 2.88)	(3.00, 2.88)
C_3	(0.50, 4.00)	(0.50, 4.00)	(0.50, 4.00)	(1.00, 4.00)	(0.50, 4.00)
$F_{C_{Z_1}}$	57.54	51.19	56.15	6.32	61.84
$F_{C_{X_2}}$	307.52	309.06	292.49	292.66	307.67
$F_{C_{Y_3}}$	122.06	122.06	113.24	118.36	111.78

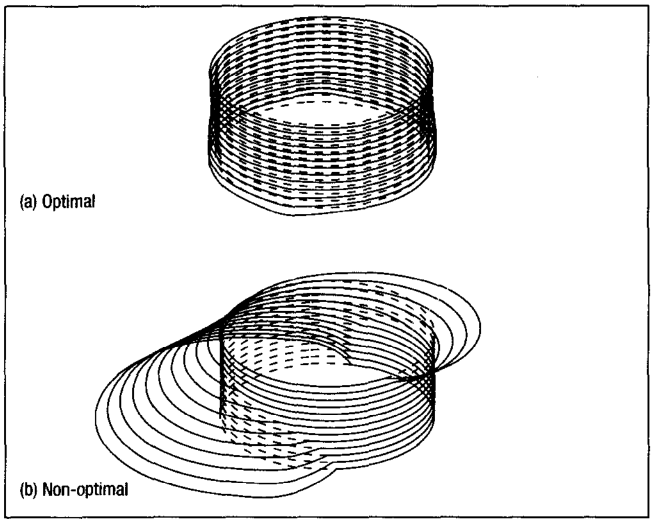


Figure 3
Isometric Views of Simulated (solid line) and Nominal (dashed line) Surfaces Generated Using (a) optimal fixture FIX1 (case I) and (b) non-optimal fixture. Error amplification factor is 250.

when $i = j$ (following the main diagonal of *Table 2*). This demonstrates that the sum of the square differences between the nominal and simulated radii throughout the drilled surface, Δ_1, is minimized when FIX1 is used. The other four fixture configurations, FIX2, FIX3, FIX4, and FIX5, result in elevated Δ_1 values. Similarly, FIX2 minimizes the maximum value of the radii square differences and FIX3 is the optimum configuration for minimizing the square differences between the nominal and simulated diameters throughout the drilled surface, whereas FIX4 minimizes the maximum value of the latter quantity. FIX5, on the other hand, minimizes the geometric deviations of the machined surface from a perfect cylindrical shape. In summary, the numerical values of the five objective functions listed in *Table 2* reflect the predicted magnitude of the geometric deviations between the simulated and nominal drilled surfaces generated by the five optimal fixture configurations. These numerical values are instrumental in evaluating the performances of different fixtures.

Isometric views of the cylindrical and simulated (shown as dashed and solid lines, respectively) hole surfaces obtained by FIX1 are shown in *Figure 3a*. *Figure 3b* depicts the same nominal and simulated hole surfaces that are obtained by using an arbitrary non-optimal fixture configuration. Notice the large deviations obtained when using a non-optimal fixture.

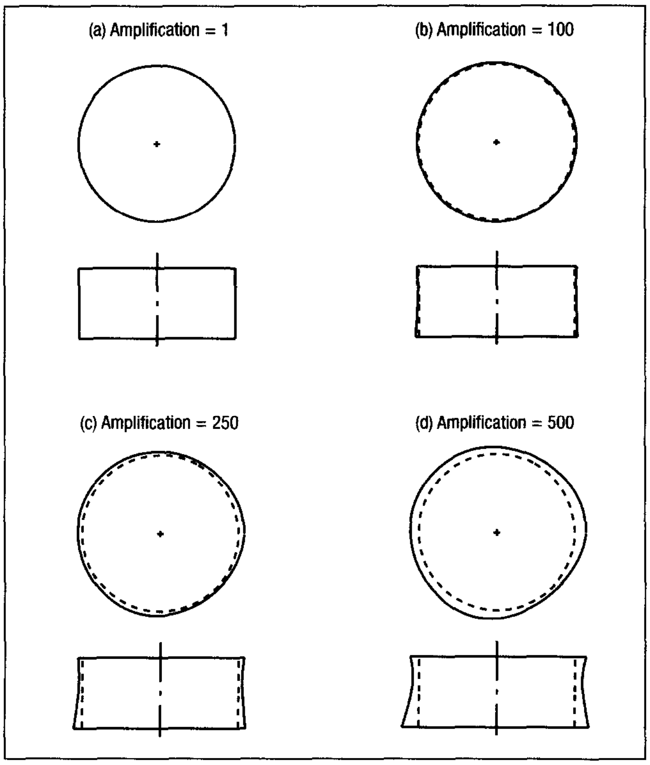


Figure 4
Effects of Different Amplification Factors of Deviations Between Simulated (solid) and Nominal (dashed) Surfaces. (a) Amplification is 1 and simulated and nominal surfaces appear to coincide. In (b), (c), and (d) the amplification factor is 100, 250, and 500. respectively. Note that an elevated amplification factor differentiates between the simulated and nominal surfaces while introducing distortions to the resultant shape. A factor of 250 is selected in this paper.

To present the simulation results, various scale factors of the deviations between the nominal and simulated hole surfaces are evaluated. *Figure 4* depicts the (*xz*) and (*xy*) views of the nominal and simulated hole surfaces when the deviation scale is magnified by a factor of 1, 100, 250, and 500. To

Table 2
Case I—Drilling 1/2 in. hole centered at (2.5, 0.5) (in.). Function values (in.2) obtained for each optimal fixture.

Function Value	FIX1	FIX2	FIX3	FIX4	FIX5
Δ_1	0.327E-08	0.539E-08	0.400E-08	0.718E-08	0.679E-08
Δ_2	0.340E-07	0.261E-07	0.467E-07	0.543E-07	0.476E-07
Δ_3	0.128E-07	0.141E-07	0.128E-07	0.194E-07	0.167E-07
Δ_4	0.858E-07	0.854E-07	0.900E-07	0.554E-07	0.896E-07
Δ_5	0.491E-09	0.569E-09	0.163E-08	0.407E-08	0.371E-09

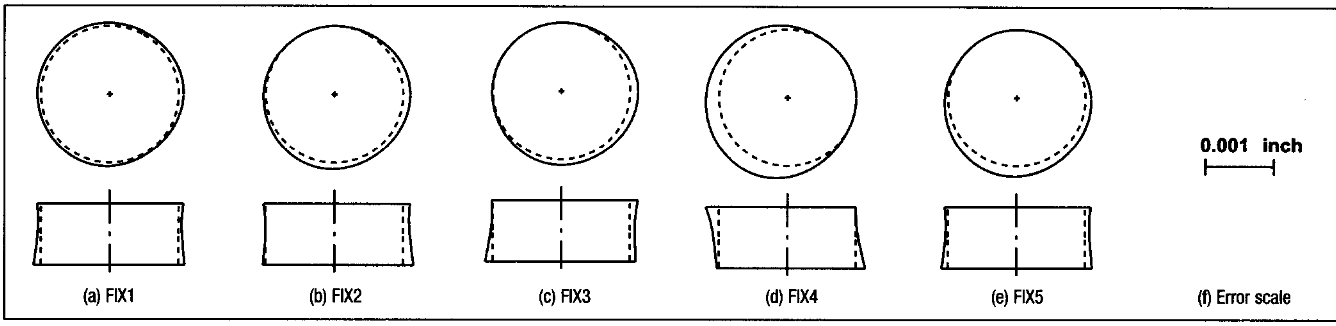
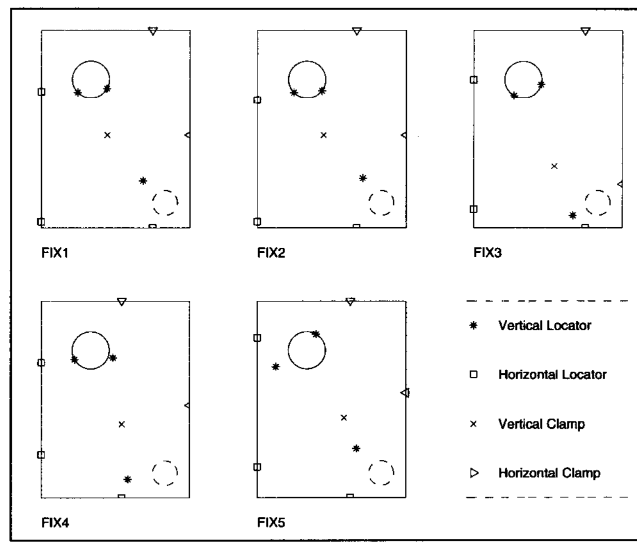


Figure 5 & 6
Case I, Side and Top Views of Nominal (dashed line) and Simulated (solid line) Drilled Surfaces Generated by (a) FIX1, (b) FIX2, ..., (e) FIX5. Hole diameter is 1/2 in.; center located at (2.5, 0.5).

capture a thousandth of an inch deviation size while keeping the effects of shape distortion manageable, a scale factor of 250 is selected for presenting the numerical drilling simulations in this paper. Side and top views of the nominal (dashed line) and simulated (solid line) drilled surfaces generated by FIX1, FIX2, ..., and FIX5 are shown in *Figure 5*. Notice the high accuracy of the simulated drilled surfaces. The deviations between the simulated and nominal hole surfaces in case I are on the order of 0.0001 in.

Figure 6 depicts the optimal fixture configurations of case II. Here, a 3/4 in. hole centered at $(X, Y) = (1.0, 3.0)$ is drilled in the presence of the 1/2 in. hole drilled in case I. The drilling thrust and torque values in this case are 694 (lb) and 193 (lb-in.), respectively.[3] The values of the optimal design fixture parameters $\bar{u}$ of case II are listed in *Table 3*; the objective function values of FIX1, FIX2, ..., and FIX5 are summarized in *Table 4*. The side (xz) and top (xy) views of the deviations between the simulated and nominal drilled surfaces are shown in *Figure 7*. Here again, the deviations between the



simulated and nominal surfaces are on the order of 0.0001 in. with the exception of FIX5, which results in larger (0.0005 in.) deviation values.

In case III, the 1/2 and 3/4 in. holes drilled by using two distinct fixture configurations in cases I and II, respectively, are drilled by using a single

Table 3
Case II—Drilling 3/4 in. hole at (1.0, 3.0) (in.) in Presence of 1/2 in. hole drilled in Case I. Locator positions (in.) and clamping forces (lb) when Δ_1 - Δ_5 are minimized.

Optimization Variables	FIX1	FIX2	FIX3	FIX4	FIX5
L_1	(2.05, 0.95)	(2.12, 1.00)	(2.00, 0.25)	(1.75, 0.38)	(2.00, 1.00)
L_2	(1.33, 2.81)	(1.30, 2.77)	(1.36, 2.90)	(1.45, 2.85)	(1.19, 3.33)
L_3	(0.73, 2.73)	(0.73, 2.73)	(0.81, 2.67)	(0.67, 2.81)	(0.38, 2.67)
L_4	(0.00, 2.75)	(0.00, 2.58)	(0.00, 3.00)	(0.00, 2.75)	(0.00, 3.25)
L_5	(0.00, 0.12)	(0.00, 0.12)	(0.00, 0.38)	(0.00, 0.88)	(0.00, 0.62)
L_6	(2.25, 0.00)	(2.00, 0.00)	(2.25, 0.00)	(1.62, 0.00)	(1.88, 0.00)
C_1	(1.33, 1.88)	(1.33, 1.88)	(1.62, 1.25)	(1.62, 1.50)	(1.75, 1.62)
C_2	(3.00, 1.88)	(3.00, 1.88)	(3.00, 0.88)	(3.00, 1.88)	(3.00, 2.12)
C_3	(2.25, 4.00)	(2.00, 4.00)	(2.25, 4.00)	(1.62, 4.00)	(1.88, 4.00)
$F_{C_{Z_1}}$	319.30	321.30	333.40	462.54	4.89
$F_{C_{X_2}}$	110.34	118.92	393.40	193.23	263.19
$F_{C_{Y_3}}$	32.90	34.58	88.93	30.55	269.61

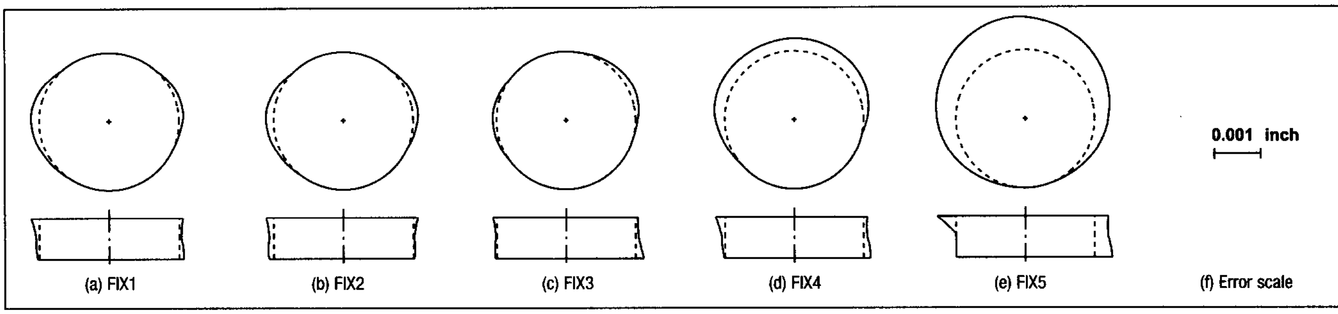


Figure 7
Case II, Side and Top Views of Nominal (dashed line) and Simulated (solid line) Drilled Surfaces Generated by (a) FIX1, (b) FIX2, ..., (e) FIX5. Hole diameter is 3/4 in.; center located at (1.0, 3.0). Hole drilled in presence of 1/2 in. hole drilled in case I.

optimal fixture. FIX1, FIX2, ..., and FIX5 shown in *Figure 8* are the optimal fixtures of case III. The values of the optimal fixture parameters $\overline{u}$ and the objective functions are listed in *Tables 5* and *6*, respectively. Note that the objective function values of case III are larger than the corresponding values calculated for cases I and II. For example, for FIX1 the values of Δ_1 are 0.327E-08 (in.^{2}), 0.366E-08 (in.^{2}), and 0.716E-08 (in.^{2}) for cases I, II, and III, respectively. This demonstrates that cases I and II result in more accurate hole surfaces when compared with the corresponding case III. Nevertheless, the fixturing setup time in case III will be doubled. The (xz) views of the deviations obtained by the five fixtures in case III are shown in *Figures 9* and *10*. Note that the deviations between the simulated and nominal surfaces in case III are of the order of 0.001 in..

In case IV, the two 1/2 and 3/4 in. holes are gang drilled. *Figure 11* depicts the optimal fixturing configurations, and *Tables 7* and *8* list the values of the corresponding optimal fixturing parameters and objective functions. Note that case IV results in elevated objective function values when compared with the corresponding values obtained in case III. For example, FIX1 results in Δ_1 values of 0.716E-08 (in.^{2}) and 0.844E-08 (in.^{2}) in cases III and IV, respectively. This implies lower accuracy of the gang drilling when compared to a sequential process such as the one simulated in case III. The (xy) and (xz) views of the deviations obtained by the five fixtures in case IV are shown in *Figures 12* and *13*.

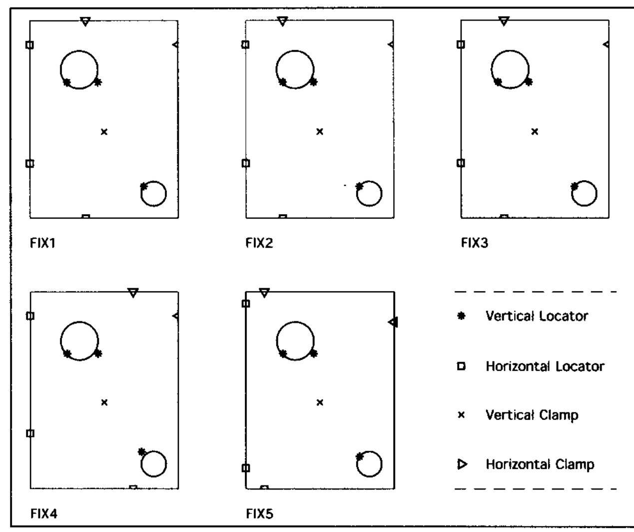


Figure 8
Optimal Fixture Configurations FIX1, FIX2, ..., FIX5 Generated Through Minimization of Δ_1, Δ_2, ..., Δ_5, Respectively, for Drilling 1/2 and 3/4 in. Holes of Case III. The two holes are drilled with a single fix-

3. Discussion

This paper introduces a linear finite element algorithm that simulates the drilling process and calcu-

Table 4
Case II—Drilling 3/4 in. Hole at (1.0, 3.0) (in.) in Presence of 1/2 in. hole drilled in Case I. Function values (in.^{2}) obtained for each optimal fixture.

Function Value	FIX1	FIX2	FIX3	FIX4	FIX5
Δ_1	0.366E-08	0.432E-08	0.421E-08	0.124E-07	0.105E-06
Δ_2	0.513E-07	0.277E-07	0.974E-07	0.749E-07	0.526E-06
Δ_3	0.129E-07	0.145E-07	0.114E-07	0.288E-07	0.231E-06
Δ_4	0.767E-07	0.925E-07	0.101E-06	0.754E-07	0.526E-06
Δ_5	0.248E-08	0.248E-08	0.130E-08	0.866E-09	0.120E-09

Table 5
Case III—Drilling 1/2 and 3/4 in. holes drilled in cases I and II, respectively, using single fixture configuration. Locator positions (in.) and clamping forces (lb) when Δ_1 - Δ_5 are minimized.

Optimization Values	FIX1	FIX2	FIX3	FIX4	FIX5
L_1	(2.30, 0.65)	(2.30, 0.65)	(2.30, 0.65)	(2.26, 0.74)	(2.30, 0.65)
L_2	(1.38, 2.75)	(1.38, 2.75)	(1.38, 2.75)	(1.38, 2.75)	(1.38, 2.75)
L_3	(0.75, 2.75)	(0.75, 2.75)	(0.75, 2.75)	(0.75, 2.75)	(0.75, 2.75)
L_4	(0.00, 3.50)	(0.00, 3.50)	(0.00, 3.50)	(0.00, 3.50)	(0.00, 3.75)
L_5	(0.00, 1.12)	(0.00, 1.12)	(0.00, 1.12)	(0.00, 1.12)	(0.00, 0.42)
L_6	(1.12, 0.00)	(0.75, 0.00)	(0.88, 0.00)	(2.08, 0.00)	(0.38, 0.00)
C_1	(1.50, 1.75)	(1.50, 1.75)	(1.50, 1.75)	(1.50, 1.75)	(1.50, 1.75)
C_2	(3.00, 3.50)	(3.00, 3.50)	(3.00, 3.50)	(3.00, 3.50)	(3.00, 3.38)
C_3	(1.12, 4.00)	(0.75, 4.00)	(0.88, 4.00)	(2.08, 4.00)	(0.38, 4.00)
$F_{C_{Z_1}}$	311.61	272.55	272.55	328.91	291.67
$F_{C_{X_2}}$	84.87	89.09	84.72	81.83	63.97
$F_{C_{Y_3}}$	5.52	5.46	2.81	34.56	82.91

Table 6
Case III—Drilling 1/2 and 3/4 in. holes drilled in cases I and II, respectively, using single fixture configuration. Function values (in.^{2}) obtained for each optimal fixture.

Function Value	FIX1	FIX2	FIX3	FIX4	FIX5
Δ_1	0.694E-08	0.752E-08	0.722E-08	0.137E-07	0.131E-07
Δ_2	0.157E-06	0.107E-06	0.108E-06	0.179E-06	0.211E-06
Δ_3	0.202E-07	0.199E-07	0.196E-07	0.312E-07	0.309E-07
Δ_4	0.432E-06	0.418E-06	0.417E-06	0.183E-06	0.426E-06
Δ_5	0.353E-08	0.377E-08	0.379E-08	0.918E-08	0.223E-08

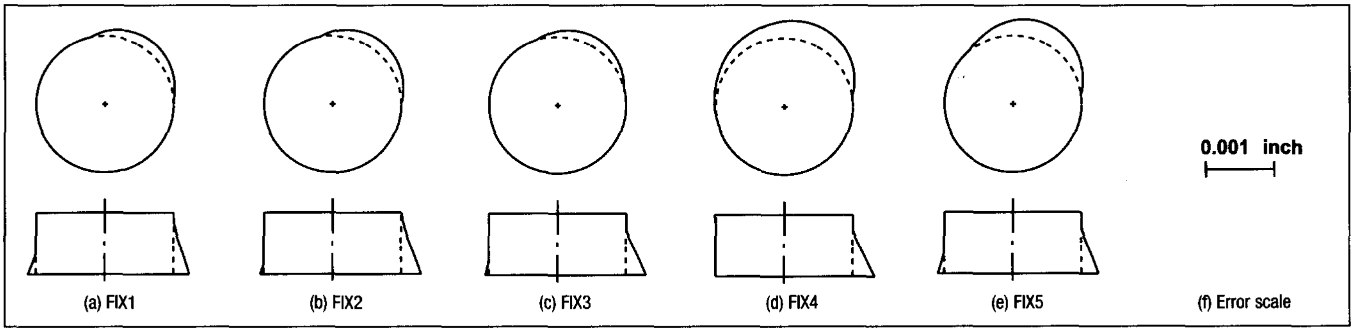


Figure 9
Case III, Sequential Drilling with a Single Fixturing Setup. Side and top views of nominal (dashed line) and simulated (solid line) drilled surfaces generated by (a) FIX1, (b) FIX2, ..., (e) FIX5. Hole diameter 1/2 in.; center located at (2.5, 0.5).

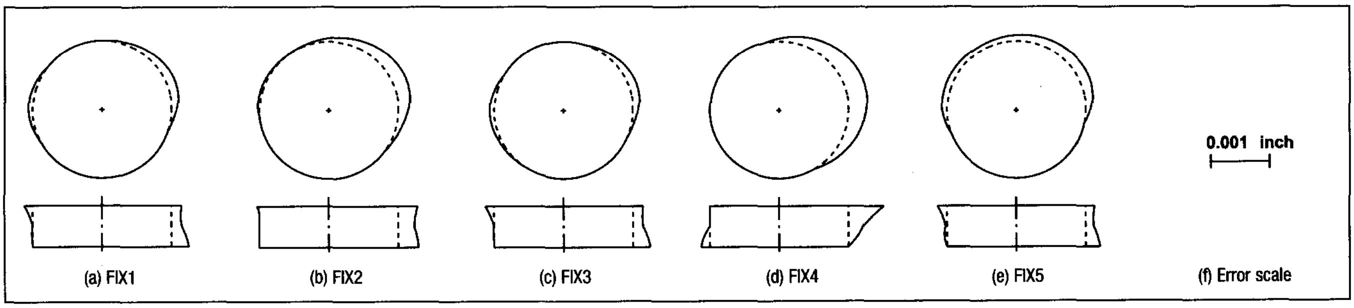


Figure 10
Case III, Sequential Drilling with a Single Fixturing Setup. Side and top views of nominal (dashed line) and simulated (solid line) drilled surfaces generated by (a) FIX1, (b) FIX2, ..., (e) FIX5. Hole diameter is 3/4 in.; center located at (1.0, 3.0). Hole drilled in presence of 1/2 in. hole centered at (2.5, 0.5) using a single fixture configuration.

Table 7
Case IV—Simultaneous drilling of 1/2 and 3/4 in. holes drilled in cases I and II. Locator positions (in.) and clamping forces (lb) when Δ_1 - Δ_5 are minimized.

Optimization Variables	FIX1	FIX2	FIX3	FIX4	FIX5
L_1	(2.30, 0.65)	(2.30, 0.65)	(2.30, 0.65)	(2.35, 0.30)	(2.30, 0.65)
L_2	(1.45, 2.85)	(2.83, 2.12)	(1.45, 2.85)	(2.42, 1.25)	(2.83, 2.25)
L_3	(0.67, 3.19)	(0.70, 3.23)	(0.67, 3.19)	(0.77, 3.30)	(0.62, 3.00)
L_4	(0.00, 3.62)	(0.00, 3.62)	(0.00, 3.62)	(0.00, 3.62)	(0.00, 3.88)
L_5	(0.00, 0.58)	(0.00, 0.58)	(0.00, 0.58)	(0.00, 1.38)	(0.00, 0.42)
L_6	(2.58, 0.00)	(2.75, 0.00)	(2.58, 0.00)	(1.17, 0.00)	(1.00, 0.00)
C_1	(2.00, 1.12)	(2.00, 1.50)	(2.00, 1.12)	(1.65, 1.58)	(2.00, 1.38)
C_2	(3.00, 3.62)	(3.00, 3.62)	(3.00, 3.62)	(3.00, 3.50)	(3.00, 3.75)
C_3	(2.58, 4.00)	(2.75, 4.00)	(2.58, 4.00)	(1.17, 4.00)	(1.00, 4.00)
$F_{C_{Z_1}}$	363.36	289.28	380.74	54.80	426.78
$F_{C_{X_2}}$	94.71	96.02	94.71	141.61	116.84
$F_{C_{Y_3}}$	13.05	17.22	13.05	4.85	58.31

Table 8
Case IV—Simultaneous drilling of 1/2 and 3/4 in. holes drilled in cases I and I. Function values (in.^{2}) obtained for each optimal fixture.

Function Value	FIX1	FIX2	FIX3	FIX4	FIX5
Δ_1	0.844E-08	0.124E-07	0.855E-08	0.159E-07	0.168E-07
Δ_2	0.298E-06	0.186E-06	0.263E-06	0.352E-06	0.210E-06
Δ_3	0.210E-07	0.359E-07	0.210E-07	0.408E-07	0.452E-07
Δ_4	0.629E-06	0.526E-06	0.641E-06	0.368E-06	0.577E-06
Δ_5	0.107E-07	0.127E-07	0.110E-07	0.132E-07	0.609E-08

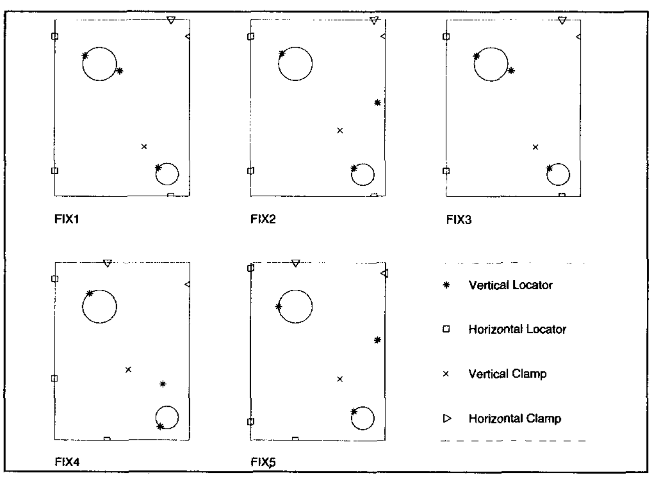


Figure 11
Optimal Fixture Configurations FIX1, FIX2, ..., FIX5 Generated Through Minimization of Δ_1, Δ_2, ..., Δ_5, Respectively, for Drilling a 1/2 in. Hole of Case IV. The two holes are drilled simultaneously.

lates the shape of the machined surface. In these simulations, the workpiece is assumed to be deformable, while the drill bit and fixture are rigid. These calculations result in the shape and dimensions of the simulated machined surface. By formulating an optimization scheme, one can obtain the fixturing layout that possesses desired characteristics. This paper introduces five optimal fixturing formulations in which one seeks the minima of five objective functions that reside inside the domain defined by the physical constraints that the fixture introduces. These constraints include static and resting equilibrium conditions, non-negative reaction forces at the locators, and non-interference with the drilling process. The latter condition prevents a locator or vertical clamp from being placed in the drilling region. Furthermore, to avoid undesired plastic deformations, the clamping forces are bounded.

The five selected objective functions capture the quadratic differences between the nominal and simulated radii (Δ_1) and diameters (Δ_3) as they are summed over the entire digitized machined surface. The maximum differences between the nominal and simulated radii and diameters are expressed by Δ_2 and Δ_4, respectively, and the deviation of the simulated machined surface from a perfect cylindrical shape is obtained by Δ_5.

The fixtures FIX1, FIX2, ..., and FIX5 (shown in *Figures 2, 6, 8,* and *11*) are the optimal fixture layouts that correspond to the minimization of Δ_1, Δ_2, ..., Δ_5, respectively. The values of the corresponding fixture design variables are listed in *Tables 1, 3, 5,* and *7*. In the simulation tests, four different drilling scenarios are evaluated. In case I, the optimal fixture for drilling a 1/2 in. hole centered at (2.5, 0.5) (in.) is sought. Case II seeks the optimal fixture layout for drilling a 3/4 in. hole at (1.0, 3.0) (in.) in the pres-

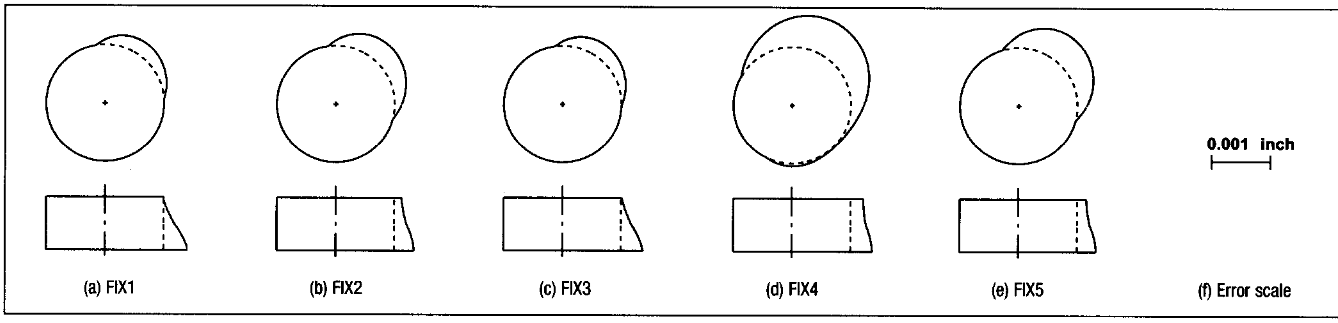


Figure 12
Case IV, Gang Drilling. Side and top views of nominal (dashed line) and simulated (solid line) drilled surfaces generated by (a) FIX1, (b) FIX2, ..., (e) FIX5. Hole diameter is 1/2 in.; center located at (2.5, 0.5).

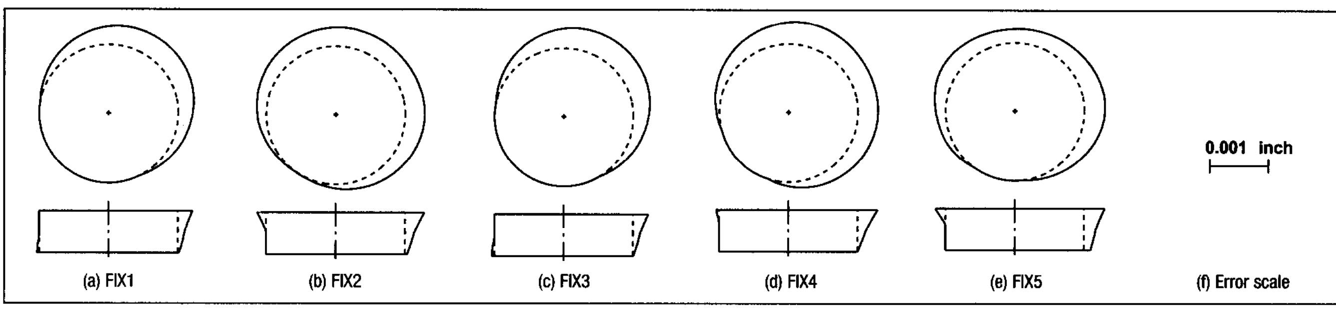


Figure 13
Case IV, Gang Drilling. Side and top views of nominal (dashed line) and simulated (solid line) drilled surfaces generated by (a) FIX1, (b) FIX2, ..., (e) FIX5. Hole diameter is 3/4 in.; center located at (1.0, 3.0).

ence of the 1/2 in. hole drilled in case I. In case III, the 1/2 and 3/4 in. holes drilled in cases I and II, respectively, are drilled by rather using a single fixture layout. This reduces the setup time by 50%, yet the machining accuracy is evaluated. In case IV, the 1/2 and 3/4 holes are gang drilled. Here, the machining time is shortened, yet the workpiece is exposed to the high loads, and the machined surface is expected to be of the lowest accuracy. *Figures 5, 7, 9, 10, 12,* and *13* depict the side and top views of the nominal and simulated drilled surfaces. In these figures, the deviations are amplified by a factor of 250 and the deviation scale of 0.001 in. is depicted. *Table 9* summarizes the numerical objective function values for the four different drilling scenarios. Note that the overall accuracy of drilling 1/2 and 3/4 in. holes by using the fixture layouts generated in case I and case II turns out to be 0.347E-08 (in.^{2}), when Δ_1 is used as a measure. If, on the other hand, these two holes are sequentially drilled by using a single fixture (case III), the Δ_1 accuracy measure turns out to be 0.694E-08 (in.^{2}), which reflects a reduction by a factor of two in the machined surface accuracy. A simulation of the gang drilling (case IV) results in a Δ_1 value of 0.844E-08 (in.^{2}), which is a reduction in accuracy of 20% when compared with the sequential drilling simulated in case III. *Table 9* summarizes the increases in error measures when sequential drilling is performed by using one vs. two fixturing setups and $\Delta_1, \Delta_2, ..., \Delta_5$ error measures are used. The effects of gang drilling are further explored, and the error measures associated with gang vs. sequential drilling are tabulated as well.

These results do reflect the known fact that one has to invest in longer setup and machining times to obtain higher accuracies. Nevertheless, the numerical formulation presented in this paper enables one to assess the increased value of the workpiece accuracy based on a user's selected measure. The authors have selected Δ_1, Δ_2, ..., and Δ_5 as accuracy measures, the correlation of which to real industrial specifications is intuitive but yet still needs to be examined.

4. Conclusions

An integrated analysis and fixture optimization model has been developed for the study of optimal

Table 9
Error Measures of Machine Surface

	Drilling 2 holes using two fixtures (sequential/2 setups) (in.^{2})	Drilling 2 holes using a single fixture (sequential/2 setups) (in.^{2})	Gang drill of 2 holes (gang/1 setup) (in.^{2})	Error ratio: two vs. one setup of sequential	Error ratio: Sequential vs. Gang using 1 setup
Δ_1	0.347E-08	0.694E-08	0.844E-08	1 : 2.0	1 : 1.2
Δ_2	0.277E-07	0.107E-06	0.186E-06	1 : 3.9	1 : 1.7
Δ_3	0.121E-07	0.196E-07	0.210E-07	1 : 1.6	1 : 1.1
Δ_4	0.754E-07	0.183E-06	0.368E-06	1 : 2.4	1 : 2.0
Δ_5	0.245E-09	0.223E-08	0.609E-08	1 : 9.1	1 : 2.7

drilling through deformable workpieces. The model makes the fundamental assumption of linear drilling and thus neglects potential nonlinearities induced by geometric changes, material damage evolution, and drill bit/workpiece interaction. Computer simulations conducted for drilling of two holes have been conducted. The associated optimal fixture configurations obtained for five objective functions have been presented. Comparisons between shape, size, and location of the drilled hole to those of the nominal one suggest that the optimal fixturing model developed herein can lead to appreciable control over the drilling process, resulting in marked improvements regarding the quality and accuracy of drilling. Independent experimental validation of the model predictions presented in this paper is planned to be conducted and may point to further model refinements. In parallel, other more physically realistic objective functions are being explored as needed to better control the hole shape, dimensions, and global positioning when drilling through deformable workpieces.

Acknowledgment

Support for P.G.C. was provided by the National Science Foundation through a Presidential Young Investigator award, grant CMS94-96209. The authors gratefully acknowledge many helpful discussions with Professor A. Belegundu on the use of optimization algorithms.

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